## International GCSE MATHEMATICS

FORMULAE SHEET - HIGHER TIER

Pythagoras'

$a^{2}+b^{2}=c^{2}$

Volume of cone $=\frac{1}{3} \pi r^{2} h$
Curved surface area of cone $=\pi r l$


Volume of sphere $=\frac{4}{3} \pi r^{3}$
Surface area of sphere $=4 \pi r^{2}$



$$
\begin{aligned}
& \text { adj }=\text { hyp } \times \cos \theta \\
& \text { opp }=\text { hyp } \times \sin \theta \\
& \text { opp }=\operatorname{adj} \times \tan \theta
\end{aligned}
$$

In any triangle $A B C$

$$
\text { or } \quad \sin \theta=\frac{\text { opp }}{\text { hyp }}
$$

$$
\cos \theta=\frac{\text { adj }}{\text { hyp }}
$$



$$
\tan \theta=\frac{\text { opp }}{\mathrm{adj}}
$$

Sine rule: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
Cosine rule: $a^{2}=b^{2}+c^{2}-2 b c \cos A$


Volume of prism $=$ area of cross section $\times$ length


Circumference of circle $=2 \pi r$
Area of circle $=\pi r^{2}$

## Area of a trapezium $=\frac{1}{2}(a+b) h$



The Quadratic Equation The solutions of $a x^{2}+b x+c=0$, where $a \neq 0$, are given by

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

## Answer ALL TWENTY TWO questions.

## Write your answers in the spaces provided.

You must write down all stages in your working.
1 The table shows information about the number of goals scored in each of the 25 matches in a hockey tournament.

| Number of goals | Number of matches |
| :---: | :---: |
| 1 | 6 |
| 2 | 8 |
| 3 | 7 |
| 4 | 3 |
| 5 | 1 |

Work out the mean number of goals.

2 The ratio of Mark's age to Reeta's age is 3:5
Mark's age is 24 years.
(a) Work out Reeta's age.

The ratio of John's age to Zahra's age is $1: 4$
The sum of their ages is 45 years.
(b) Work out Zahra's age.


Diagram NOT
accurately drawn
$x=$

Diagram NOT
accurately drawn

The diagram shows a regular 6-sided polygon.
(b) Work out the value of $y$.


$$
y=
$$

4 (a) Factorise $t^{2}+6 t$
(b) Solve $7 x-5=5 x-4$

Show clear algebraic working.
$x=$
(3)
(c) Expand and simplify fully $4(2 y+3)+2(y-6)$
$5 \mathscr{E}=\{$ even numbers $\}$
$A=\{$ factors of 8$\}$
$B=\{$ factors of 20$\}$
List the members of $A \cap B$

Do NOT write in this space.

6 (a) Dilip buys a painting for $\$ 675$
Later, he sells it and makes a percentage profit of $12 \%$.
Work out the price for which Dilip sells the painting.
(b) Renuka sells her car.

She makes a loss of \$2162
Her percentage loss is $23 \%$.
Work out the price for which Renuka sells her car.
(c) Lin bought a computer that had a value of $\$ 1500$

At the end of each year, the value of her computer had depreciated by $40 \%$ of its value at the start of that year.

Calculate the value of her computer at the end of 3 years.


Diagram NOT
accurately drawn

The width of the rectangle is 64.8 cm and the height is 48.6 cm .
The length of a diagonal of the rectangle gives the 'size' of the TV screen.
(a) Calculate the 'size' of the TV screen.

38.4 cm


Diagram NOT
accurately drawn

The diagram shows two rectangular TV screens.
The rectangles are similar.
The 'size' of the smaller screen is 48 cm .
The width of the smaller screen is 38.4 cm .
The 'size' of the larger screen is 102 cm .
(b) Calculate the width of the larger TV screen.

8 Morse Code uses dots ( $\boldsymbol{\bullet}$ ) and dashes ( $\boldsymbol{-}$ ) to represent each letter of the alphabet.
Here are 10 cards.
Each card has the Morse Code for a letter on it.

(a) Kelly takes at random one of the cards.

Find the probability that she takes a card with 2 dots or a card with 3 dots.
(b) Hashim has the 10 cards.

He takes at random a card 200 times.
He replaces the card each time.
Work out an estimate for the number of times he will take a card with exactly 2 dots.
(c) Shani takes at random two of the 10 cards without replacement.

Calculate the probability that
(i) there is exactly 1 dot on each card she takes,
(ii) there is a total of 4 dots on the two cards she takes.

9 (a) Simplify $\frac{y^{8}}{y^{3}}$
(b) Solve the inequality $4(x+3)>8$

10 The grouped frequency table gives information about the lengths of time 160 students exercised one day.

| Time ( $t$ minutes) | Frequency |
| :---: | :---: |
| $0<t \leqslant 40$ | 20 |
| $40<t \leqslant 80$ | 35 |
| $80<t \leqslant 120$ | 60 |
| $120<t \leqslant 160$ | 33 |
| $160<t \leqslant 200$ | 7 |
| $200<t \leqslant 240$ | 5 |

(a) Complete the cumulative frequency table.

| Time ( $t$ minutes) | Cumulative <br> frequency |
| :---: | :---: |
| $0<t \leqslant 40$ |  |
| $0<t \leqslant 80$ |  |
| $0<t \leqslant 120$ |  |
| $0<t \leqslant 160$ |  |
| $0<t \leqslant 200$ |  |
| $0<t \leqslant 240$ |  |

(b) On the grid, draw a cumulative frequency graph for your table.

Cumulative frequency

(2)
(c) Use your graph to find an estimate for the lower quartile of the lengths of time the 160 students exercised.
minutes

11 Find the Lowest Common Multiple (LCM) of 20 and 24

12


Triangle $P Q R$ is an enlargement, centre $O$, of triangle $A B C$.
$O A P$ and $O B Q$ are straight lines.
$O A=2 \mathrm{~cm}$.
$A P=6 \mathrm{~cm}$.
$B Q=7.2 \mathrm{~cm}$.
$A C=3.7 \mathrm{~cm}$.
(a) Work out the length of $O B$.
(b) Work out the length of $P R$.

The area of triangle $P Q R$ is $72 \mathrm{~cm}^{2}$
(c) Work out the area of triangle $A B C$.

13 (a) Solve the simultaneous equations $3 x+5 y=14$

$$
4 x+3 y=4
$$

Show clear algebraic working.

$$
x=
$$

$\qquad$
(b) Write down the coordinates of the point of intersection of the two lines whose equations are $3 x+5 y=14$ and $4 x+3 y=4$
(.. $\qquad$


Diagram NOT accurately drawn

The diagram shows a shape made from a solid cube and a solid cylinder.
The cube has sides of length 8.7 cm .
The cylinder has a radius of 2.7 cm and a height of 4.9 cm .
Calculate the total surface area of the solid shape.
Give your answer correct to 3 significant figures.

15 A particle moves along a straight line.
The fixed point $O$ lies on this line.
The displacement of the particle from $O$ at time $t$ seconds is $s$ metres, where

$$
s=t^{3}-6 t+3
$$

(a) Find an expression for the velocity, $v \mathrm{~m} / \mathrm{s}$, of the particle at time $t$ seconds.

$$
v=
$$

(b) Find the acceleration of the particle at time 5 seconds.
$\qquad$ $\mathrm{m} / \mathrm{s}^{2}$

16 Make $r$ the subject of the formula $A=4 r^{2}-\pi r^{2}$ where $r$ is positive.

## Do NOT write in this space.

17


Diagram NOT accurately drawn

The diagram shows triangle $A B C$.
$D$ is the point on $A B$, such that $C D$ is perpendicular to $A B$.
$A C=8.3 \mathrm{~cm}$.
$A D=4.7 \mathrm{~cm}$.
$B D=7.5 \mathrm{~cm}$.
Calculate the size of angle $A B C$.
Give your answer correct to 1 decimal place.

18


Ivan is a shot putter.
The formula $h=2+6 t-5 t^{2}$ gives the height, $h$ metres, of the shot above the ground $t$ seconds after he has released the shot.
(i) Solve $2+6 t-5 t^{2}=0$

Give your solutions correct to 3 significant figures.
Show your working clearly.

The shot hits the ground after $T$ seconds.
(ii) Write down the value of $T$.

Give your answer correct to 3 significant figures.
$\qquad$

19 Given that $x$ and $y$ are positive integers such that $(1+\sqrt{x})(3+\sqrt{x})=y+4 \sqrt{5}$
find the value of $x$ and the value of $y$.
$x=$
$y=$

20 Simplify fully $\frac{x^{2}-16}{x^{2}-6 x+8}$

21


The diagram shows a regular pentagon inside a circle, centre $O$.
The points $A$ and $B$ lie on the circle such that $A B$ is a side of the pentagon.
$O A=7 \mathrm{~cm}$.
$T A$ is a tangent to the circle and $O B T$ is a straight line.
Calculate the area of triangle $A B T$.
Give your answer correct to 3 significant figures.

22 The functions f and g are such that $\mathrm{f}(x)=x+3 \quad$ and $\quad \mathrm{g}(x)=\frac{1}{x-2}$
(a) Find $\operatorname{fg}(x)$

Give your answer as a single algebraic fraction expressed as simply as possible.
(b) Express the inverse function $\mathrm{g}^{-1}$ in the form $\mathrm{g}^{-1}(x)=\ldots$

$$
\mathrm{g}^{-1}(x)=
$$

$\qquad$

