International GCSE MATHEMATICS
FORMULAE SHEET – HIGHER TIER

Pythagoras' Theorem
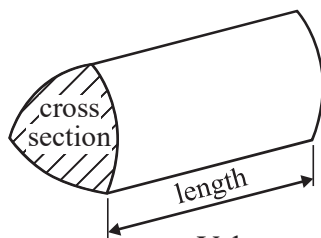
$$a^2 + b^2 = c^2$$

Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$
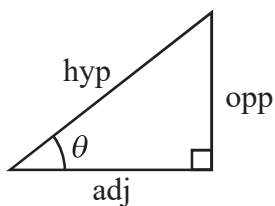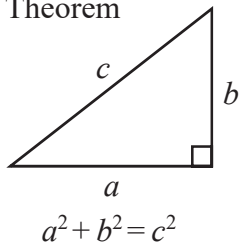
adj $=$ hyp $\times \cos\theta$
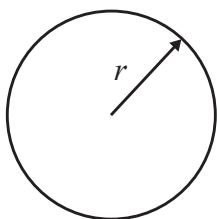opp $=$ hyp $\times \sin\theta$
opp $=$ adj $\times \tan\theta$

or

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$
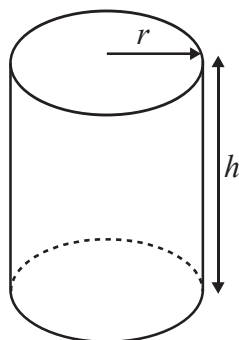
$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

In any triangle ABC

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle $= \frac{1}{2}ab\sin C$

Volume of prism $=$ area of cross section $\times$ length

Circumference of circle $= 2\pi r$

Area of circle $= \pi r^2$

Area of a trapezium $= \frac{1}{2}(a + b)h$

Volume of cylinder $= \pi r^2 h$

Curved surface area of cylinder $= 2\pi rh$
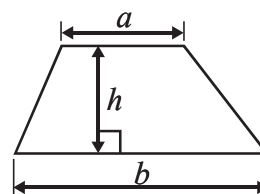
The Quadratic Equation
The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Answer ALL TWENTY TWO questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1   Rafael and Roger played tennis against each other 30 times.
Each of the times they played, either Rafael won or Roger won.
The ratio of the number of times Rafael won to the number of times Roger won is $7:3$

(a) Work out the number of times Rafael won.

....

(2)

In a school, there are 75 girls in the tennis squad.
The ratio of the number of boys in the tennis squad to the number of girls in the tennis squad is $4:3$

(b) Work out the number of boys in the tennis squad.

....

(2)

(Total for Question 1 is 4 marks)

2 (a) Factorise fully $2x^2 - 4x$

....

(2)

$A = 2p + 3q$

(b) Work out the value of p when $A = 32$ and $q = 7$

$p = $ ....

(3)

(Total for Question 2 is 5 marks)

3 There are 50 marbles in a bag.
35 of the marbles are brown.

Otti takes at random a marble from the bag.
He records the colour of the marble and puts the marble back in the bag.

He does this 300 times.

Work out an estimate for the number of brown marbles he takes.

....

(Total for Question 3 is 2 marks)

4 Work out the size of an exterior angle of a regular polygon with 8 sides.

.... °

(Total for Question 4 is 2 marks)

5 In a sale, normal prices are reduced by 8%

(a) The normal price of a jacket is £28

Work out the price of the jacket in the sale.

£ ....
(3)

(b) In the sale, the price of a shirt decreases by £3

Work out the normal price of the shirt.

£ ....
(3)

(Total for Question 5 is 6 marks)

6 (a) Solve the inequalities $-4 < 3x + 5 \leqslant 11$

....

(3)

(b) Write down the integer values of x which satisfy $-4 < 3x + 5 \leqslant 11$

....

(2)

(Total for Question 6 is 5 marks)

7 Write 792 as a product of its prime factors.
Show your working clearly.

....

(Total for Question 7 is 3 marks)

P 4 5 8 6 4 A 0 6 2 0

8
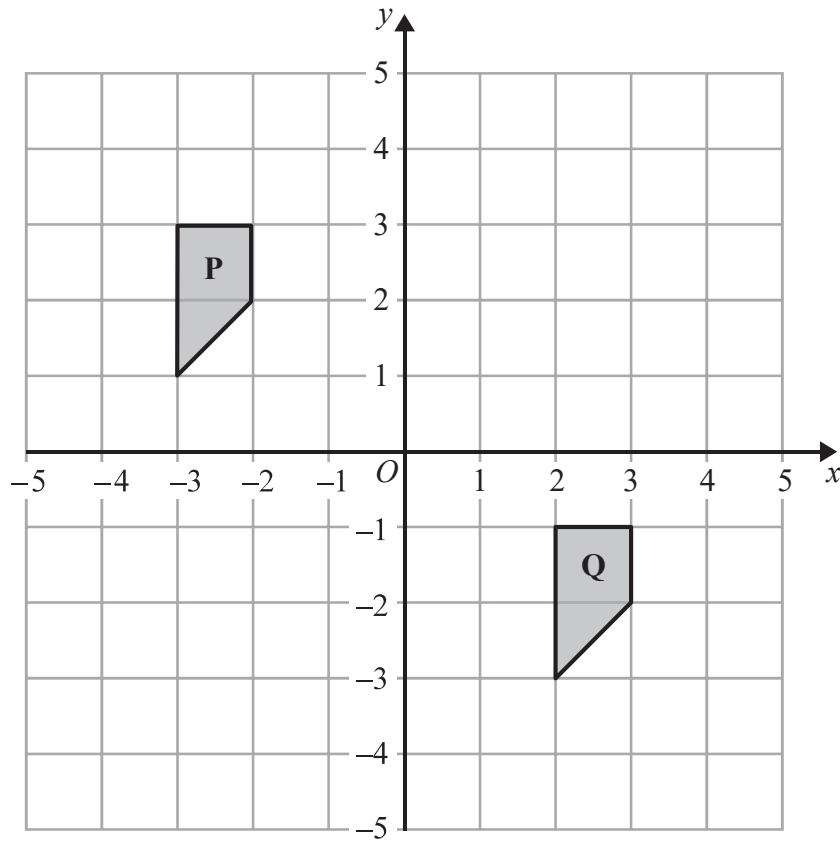
(a) Describe fully the single transformation that maps shape **P** onto shape **Q**.

....

....

(2)

(b) Rotate shape **Q** 90° clockwise about (1,0)
Label the new shape **R**.

(2)

(Total for Question 8 is 4 marks)

9 Li throws a 6-sided biased dice once.

The table shows the probability that the dice will land on 1, 2, 3, 5 or 6

Number	1	2	3	4	5	6
Probability	0.15	0.1	0.05		0.2	0.15

(a) Work out the probability that the dice will land on 4

....

(2)

(b) Work out the probability that the dice will land on an odd number.

....

(2)

(Total for Question 9 is 4 marks)

10 Julie asked 50 children how many exercise sessions they each took part in last month.
The table shows information about her results.

Number of exercise sessions	Frequency
0 to 6	13
7 to 13	10
14 to 20	16
21 to 27	7
28 to 34	4

Calculate an estimate for the total number of exercise sessions the children took part in last month.

....

(Total for Question 10 is 3 marks)

11 The line **L** passes through the point $(3, 1)$ and is parallel to the line with equation $y = \dfrac{7}{2} - 2x$.

Find an equation for the line **L**.

....

(Total for Question 11 is 3 marks)

12 (a) Simplify fully $\dfrac{a^{11}}{a^2 \times a^5}$

....

(2)

(b) Make p the subject of $p + 4q = 3p + 5$

....

(2)

(c) Expand and simplify $(2y + 3)(4y - 1)$

....

(2)

(d) Simplify $(8a^6b^3)^{\frac{1}{3}}$

....

(2)

(Total for Question 12 is 8 marks)

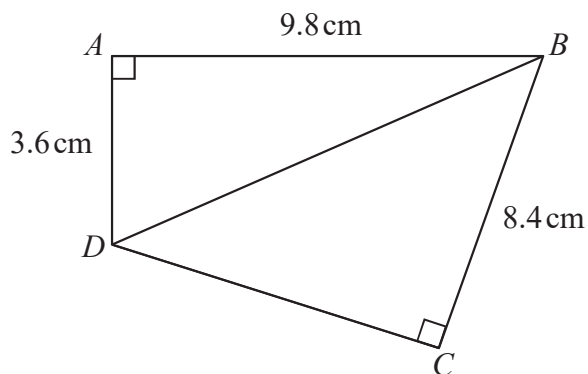**13** Here is the quadrilateral *ABCD*.

A — 9.8 cm — B

3.6 cm

8.4 cm

D

C

Diagram **NOT** accurately drawn

Angle *BAD* = 90° and angle *BCD* = 90°
AB = 9.8 cm
AD = 3.6 cm
BC = 8.4 cm

Calculate the length of *DC*.

.... cm

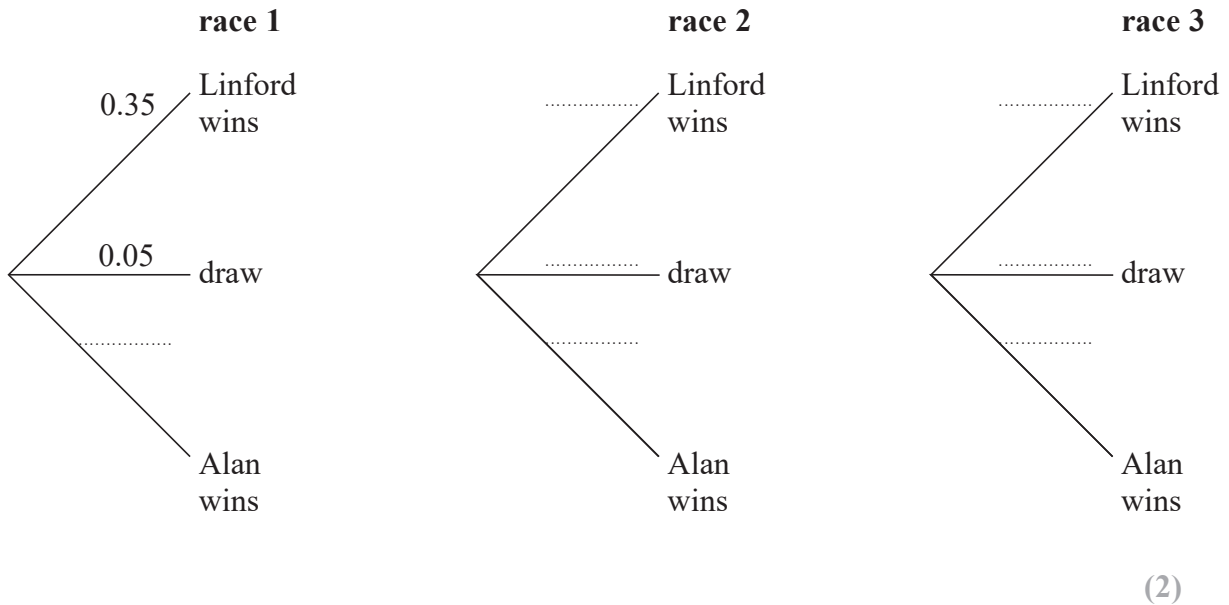**(Total for Question 13 is 4 marks)**

Turn over ▶

14 Linford and Alan race against each other in a competition.

If one of them wins a race, he wins the competition.
If the race is a draw, they run another race.

They run a maximum of three races.

Each time they race, the probability that Linford wins is 0.35
Each time they race, the probability that there is a draw is 0.05

(a) Complete the probability tree diagram.

race 1	race 2	race 3

(2)

(b) Calculate the probability that Linford wins the competition.

....

(3)

(Total for Question 14 is 5 marks)

15 $y = x^3 - \dfrac{9}{2}x^2 - 54x + 10$

(a) Find $\dfrac{dy}{dx}$

....

(2)

The curve with equation $y = x^3 - \dfrac{9}{2}x^2 - 54x + 10$ has two turning points.

(b) Find the x coordinate of each of these two points.

....

(3)

(Total for Question 15 is 5 marks)

16 The incomplete histogram shows information about the heights of a group of children.
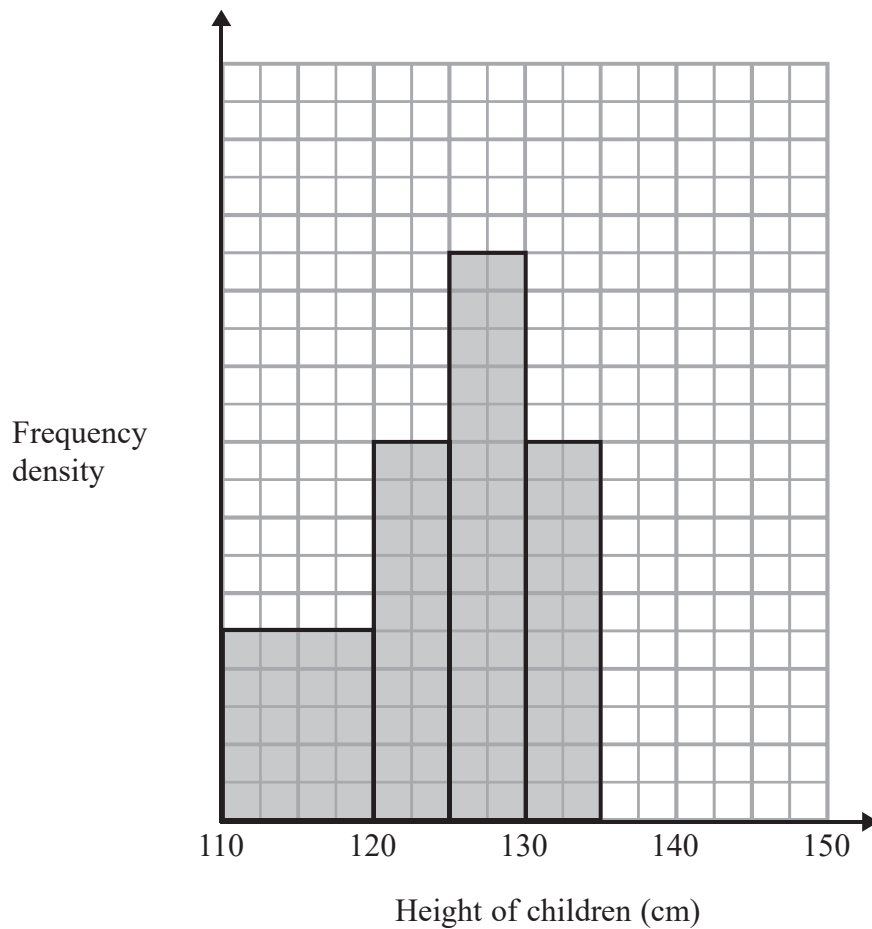
There were 10 children with heights between 130 cm and 135 cm.

(a) How many children had heights between 110 cm and 130 cm?

....

(3)

There were 6 children with heights between 135 cm and 145 cm.

(b) Show this information on the histogram.

(1)

(Total for Question 16 is 4 marks)

17

Diagram **NOT** accurately drawn
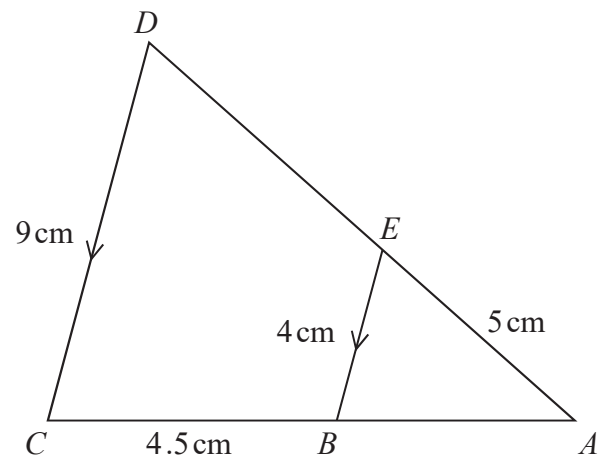
Triangle *ABE* is similar to triangle *ACD*.
AED and *ABC* are straight lines.
EB and *DC* are parallel.
AE = 5 cm, *BC* = 4.5 cm, *BE* = 4 cm, *CD* = 9 cm

(a) Calculate the length of *AD*.

.... cm
(2)

(b) Calculate the length of *AB*.

.... cm
(2)

The area of quadrilateral *BCDE* is x cm^2
The area of triangle *ABE* is y cm^2

(c) Find an expression for y in terms of x.
Give your answer as simply as possible.

y = ...
(3)

(Total for Question 17 is 7 marks)

18 f is the function such that

$$f(x) = \frac{x}{3x + 1}$$

(a) Find f(0.5)

....

(1)

(b) Find ff(−1)

....

(2)

(c) Find the value of x that cannot be included in any domain of f

....

(1)

(d) Express the inverse function f⁻¹ in the form f⁻¹(x) = ...
   Show clear algebraic working.

f⁻¹(x) = ...

(3)

(Total for Question 18 is 7 marks)

19

Diagram **NOT** accurately drawn
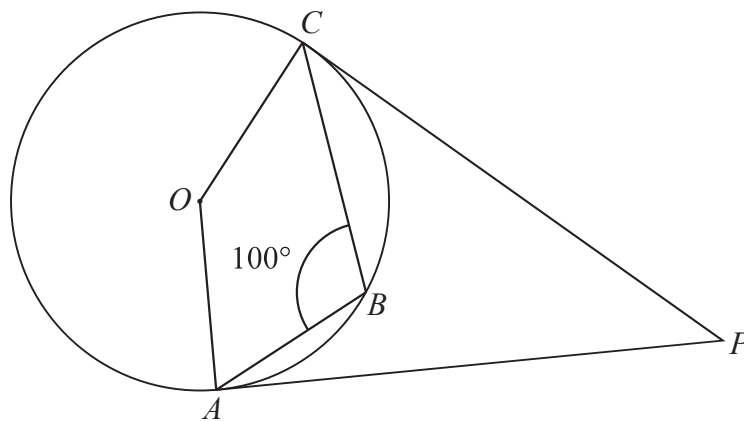
A, *B* and *C* are points on a circle, centre *O*.
PA and *PC* are tangents to the circle.
Angle *ABC* = 100°

Calculate the size of angle *APC*.

....°

(Total for Question 19 is 3 marks)

20 (a) Simplify fully $\dfrac{50x^2 - 8}{10x - 4}$

    Show clear algebraic working.

(3)

(b) Given that a is a positive integer, show that

$$\sqrt{3a}\left(\sqrt{12a} + a\sqrt{3a}\right)$$

    is always a multiple of 3

(3)

(Total for Question 20 is 6 marks)

21 Solve $3 \times 4^{2k+8} = 24$

Show your working clearly.

$k =$ ....

(Total for Question 21 is 4 marks)

22
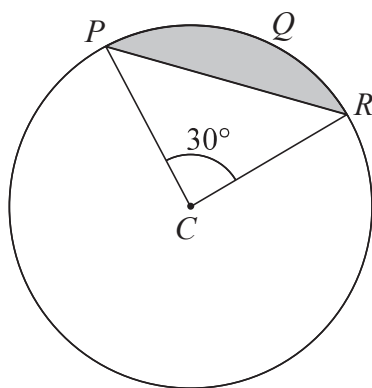
Diagram **NOT** accurately drawn

The diagram shows a circle, centre *C*.
PR is a chord of the circle.
The area of the shaded region is 100 cm²
Angle *PCR* = 30°

Calculate the length of the arc *PQR*.
Give your answer correct to 3 significant figures.

.... cm

(Total for Question 22 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS