## International GCSE MATHEMATICS <br> FORMULAE SHEET - HIGHER TIER



Volume of cone $=\frac{1}{3} \pi r^{2} h$
Volume of sphere $=\frac{4}{3} \pi r^{3}$
Curved surface area of cone $=\pi r l$


Surface area of sphere $=4 \pi r^{2}$

$\operatorname{adj}=\operatorname{hyp} \times \cos \theta$
opp $=$ hyp $\times \sin \theta$
opp $=\operatorname{adj} \times \tan \theta$
or $\sin \theta=\frac{\text { opp }}{\text { hyp }}$
$\cos \theta=\frac{\text { adj }}{\text { hyp }}$
$\tan \theta=\frac{\text { opp }}{\text { adj }}$
In any triangle $A B C$


Sine rule: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
Cosine rule: $a^{2}=b^{2}+c^{2}-2 b c \cos A$


Circumference of circle $=2 \pi r$
Area of circle $=\pi r^{2}$

$$
\text { Area of a trapezium }=\frac{1}{2}(a+b) h
$$



The Quadratic Equation
The solutions of $a x^{2}+b x+c=0$, where $a \neq 0$, are given by

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

## Answer ALL TWENTY FOUR questions.

Write your answers in the spaces provided.

## You must write down all stages in your working.

1 The ocean liner Queen Mary 2 is the longest of its type. It has a length of 345 metres.

A scale model is made of the Queen Mary 2
The scale of the model is $1: 200$
Work out the length of the scale model.
Give your answer in centimetres.


2 The pie chart gives information about the amounts spent by a gas company in one year.


Diagram NOT accurately drawn

The amount spent on materials was 225.5 million euros.
The amount spent on services was the same as the amount spent on wages.
Work out the amount spent on services.
million euros

3 The first four terms of an arithmetic sequence are

$$
\begin{array}{llll}
5 & 9 & 13 & 17
\end{array}
$$

(a) Write down an expression, in terms of $n$, for the $n$th term.
(b) Write down an expression, in terms of $n$, for the $(n+1)$ th term.
$4 w, x, y$ and $z$ are 4 integers written in order of size, starting with the smallest.
The mean of $w, x, y$ and $z$ is 13
The sum of $w, x$ and $y$ is 33
(a) Find the value of $z$.

$$
\begin{equation*}
z= \tag{2}
\end{equation*}
$$

Given also that the range of $w, x, y$ and $z$ is 10
(b) work out the median of $w, x y$ nd $z$.

5 On 1st May 2012, the cost of 5.7 grams of gold was 15960 rupees.
(a) Work out the cost, in rupees, of 4.6 grams of gold on the same day.
rupees

The cost of gold decreased by 7.5\% from 1st May 2012 to 1st May 2013
(b) Work out the cost, in rupees, of 5.7 grams of gold on 1st May 2013
rupees
(3)

6 A steam engine for pulling trains has wheels of diameter 1.5 metres.
(a) Calculate the circumference of a wheel.

Give your answer correct to 3 significant figures.


The steam engine travels 1000 metres along a test ack.
(b) Work out the number of complete turns of wheel.

7 John changes $£ 450$ to euros.
The exchange rate is $£ 1=1.16$ euros.
(a) Change $£ 450$ to euros.
euros

When in Amsterdam, John uses his credit card to pay for a ring costing 850 euros.
He has to pay a bank charge of $£ 3.50$ for using his credit card in addition to the cost of the ring.
(b) Work out the total cost, in pounds (£), of the ring and the bank charge.
$£$

8 Here is a right-angled triangle.

$A C=6.5 \mathrm{~cm}$.
$B C=6.3 \mathrm{~cm}$.
Angle $A B C=90^{\circ}$
Calculate the length of $A B$.

9 (a) Simplify $5 y \times 4 y^{2}$
(b) Simplify $\frac{15 e^{2} f}{25 e f^{3}}$
(c) Factorise $6 p^{2}-5 p q-6 q^{2}$
(d) Simplify ( ${ }^{y}$

10 The table shows some information about the five Great Lakes in North America.

| Name | Surface area $\left(\mathbf{m}^{\mathbf{2}}\right)$ | Volume of water $\left(\mathbf{m}^{3}\right)$ |
| :--- | :---: | :---: |
| Lake Erie | $2.57 \times 10^{10}$ | $4.80 \times 10^{11}$ |
| Lake Huron | $6.01 \times 10^{10}$ | $3.52 \times 10^{12}$ |
| Lake Michigan | $5.80 \times 10^{10}$ | $4.87 \times 10^{12}$ |
| Lake Ontario | $1.91 \times 10^{10}$ | $1.64 \times 10^{12}$ |
| Lake Superior | $8.21 \times 10^{10}$ | $1.22 \times 10^{13}$ |

(a) Work out the total surface area of the five Great Lakes.

Give your answer in standard form.

Loch Ness is the larges ake Scotland.
The lake has a volume ter f745 $\times 10^{9} \mathrm{~m}^{3}$
The volum of w in Lake Superio is $k$ times the volume of water in Loch Ness.
(b) Work out th value o $k$.

Give y ur ans er correct to 3 significant figures.

$$
k=
$$

11 Here is a prism.


Diagram NOT accurately drawn
$A B C D E F$ is a cross section of the prism.
$A B C F$ is a square of side 12 cm .
$F C D E$ is a trapezium.
$E D=22 \mathrm{~cm}$.
The height of the prism is 20 cm .
The length of the prism is 80 cm .
Work out the total volume of he pr m .

12 There are 32 students in Mr Newton's class.
20 are boys and 12 are girls.
The mean height of the boys is 151 cm .
The mean height of the girls is 148 cm .
Calculate the mean height of all the students in Mr Newton's class.

13 (a) Solve

$$
\begin{aligned}
& 3 x+3 y=9 \\
& 4 x+2 y=13
\end{aligned}
$$

Show clear algebraic working.

$$
\begin{aligned}
& x= \\
& y=
\end{aligned}
$$

$\mathbf{L}$ is a line parallel to th line ith equation $4 x+2 y=13$
$L$ passes ough the point th co dinates $(3,-1)$
(b) Find an equation the line $\mathbf{L}$.

14 (a) Factorise $a^{2}-b^{2}$
$N=2^{22}-1$
(b) Write $N$ as the product of two integers, both of which are greater than 1000
$15 A B C D$ is a trapezium.

$A B=25 \mathrm{~cm}$.
$B C=24 \mathrm{~cm}$.
$C D=10 \mathrm{~cm}$.
Angle $A B C=$ angle $B C D=90^{\circ}$
Calculate the size of angle $C D A$.
Give your answer correct to 3 significant figures.

16 (a) Complete the table of values for $y=\frac{1}{2}\left(x+\frac{9}{x}\right)$

| $x$ | 1 | 1.5 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 5 |  | 3.25 |  | 3.125 | 3.4 |  |

(2)
(b) Draw the graph of $y=\frac{1}{2}\left(x+\frac{9}{x}\right)$ for values of $x$ from 1 to 6

(2)
(c) Use the graph to find estimates for the solutions of the equation $x+\frac{9}{x}=7$
$17 \mathrm{f}(x)=\frac{3}{x+1}+\frac{1}{x-2}$
(a) State one value of $x$ which cannot be included in any domain of f .
(b) Find the value of $f(0)$
(c) Find the value $\mathrm{f} x$ for which $\mathrm{f}(x)=0$ Show clear algebr c wor ing.

$$
x=
$$

$18 y=\frac{2 a}{b-c}$
$a=42$ correct to 2 significant figures.
$b=24$ correct to 2 significant figures.
$c=14$ correct to 2 significant figures.
Work out the lower bound for the value of $y$.
Give your answer correct to 2 significant figures.
Show your working clearly.

19 The table gives information about the areas of some farms in France.

| Area $(\boldsymbol{A}$ hectares $)$ | Frequency |
| :---: | :---: |
| $0<A \leqslant 20$ | 50 |
| $20<A \leqslant 50$ | 90 |
| $50<A \leqslant 100$ | 120 |
| $100<A \leqslant 300$ | 160 |

On the grid, draw a histogram to show this information.

(Total for Question 19 is 3 marks)

20 Leonidas has a fair dice.
He throws the dice twice.
(a) Work out the probability that he gets the number 5 both times.


Alicia has a fair dice.
She throws the dice 3 times.
(b) Work out the probability th $t$ she gets th umber 5 exactly once.

21

$A, B$ and $C$ are three points on a circle.
$D C A$ is a straight line.
$C A$ is a diameter of the circle.
$D B$ is a tangent to the circle.
Calculate the size of angle $C D B$.

Diagram NOT accurately drawn
$22 A, r$ and $T$ are three variables.
$A$ is proportional to $T^{2}$
$A$ is also proportional to $r^{3}$
$T=47$ when $r=0.25$
Find $r$ when $T=365$
Give your answer correct to 3 significant figures.

23


Diagram NOT accurately drawn
$A B C D$ is the square base of the pyramid $V A B C D$.
$A B=B C=C D=D A=10 \mathrm{~cm}$.
$V A=V B=V C=V D=12 \mathrm{~cm}$.
Calculate the height of the pyramid.
Give your answer correct to 3 significant figures.


Diagram NOT accurately drawn

In triangle $O P Q, \overrightarrow{O P}=6 \mathbf{a}$ and $\overrightarrow{O Q}=6 \mathbf{b}$
$X$ is the midpoint of $P Q$.
(a) Find, in terms of a and $\mathbf{b}$, the vector $\overrightarrow{O X}$ Give your answer in its simplest form.
$Y$ is the point on $O X$ such $\mathrm{t} \quad \mathrm{t} O Y \quad Y X=2 \quad 1$
(b) Find, in terms of a a d $\mathbf{b}$, he vector $\overrightarrow{Q Y}$ Give your answer i imp st form.

