## International GCSE MATHEMATICS <br> FORMULAE SHEET - HIGHER TIER

Pythagoras'


$$
a^{2}+b^{2}=c^{2}
$$

Volume of cone $=\frac{1}{3} \pi r^{2} h$
Curved surface area of cone $=\pi r l$


Volume of sphere $=\frac{4}{3} \pi r^{3}$
Surface area of sphere $=4 \pi r^{2}$


adj $=$ hyp $\times \cos \theta$
opp $=$ hyp $\times \sin \theta$
opp $=\operatorname{adj} \times \tan \theta$
In any triangle $A B C$


Sine rule: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
Cosine rule: $a^{2}=b^{2}+c^{2}-2 b c \cos A$


Volume of prism $=$ area of cross section $\times$ length


Circumference of circle $=2 \pi r$
Area of circle $=\pi r^{2}$

## Area of a trapezium $=\frac{1}{2}(a+b) h$



Volume of cylinder $=\pi r^{2} h$
Curved surface area of cylinder $=2 \pi r h$

The Quadratic Equation
The solutions of $a x^{2}+b x+c=0$, where $a \neq 0$, are given by

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

## Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.

You must write down all stages in your working.
1 The table shows information about the numbers of fish caught by 40 people in one day.

| Number of fish | Frequency |
| :---: | :---: |
| 0 | 2 |
| 1 | 12 |
| 2 | 15 |
| 3 | 8 |
| 5 | 2 |
| 8 | 1 |

(a) Work out the mean number of fish caught.
(b) Work out what percentage of the 40 people caught less than 2 fish.

2 Each exterior angle of a regular polygon is $15^{\circ}$
(a) How many sides has the regular polygon?

The diagram shows 3 identical regular pentagons.

(b) Work out the value of $y$.

3 Use your calculator to work out the value of

$$
\frac{12.5 \times 4.5}{6.8+\sqrt{67.24}}
$$

4 Solve $7 x-2=1-3 x$
Show clear algebraic working.

$$
x=
$$

5

(a) Describe fully the single transformation which maps shape $\mathbf{P}$ onto shape $\mathbf{Q}$.
(b) Reflect the shape $\mathbf{Q}$ in the line $y=x$.

Label the new shape $\mathbf{R}$.
(2)

(c) Enlarge shape $\mathbf{S}$ with scale factor $\frac{1}{2}$ and centre $(1,3)$

6 The mean height of a group of 6 children is 165 cm .
One child, whose height is 155 cm , leaves the group.
Find the mean height of the remaining 5 children.

7


Diagram NOT accurately drawn

Work out the value of $x$.
Give your answer correct to 3 significant figures.

8 (a) Factorise $g^{2}+4 g$
(b) Factorise $e^{2}-2 e-24$

9 Make $r$ the subject of the formula $A=4 \pi r^{2}$ where $r$ is positive.
$r=$

10 (a) $A=2^{2} \times 3 \times 5^{2}$
$B=2^{3} \times 5$
(i) Find the Highest Common Factor (HCF) of $A$ and $B$.
(ii) Find the Lowest Common Multiple (LCM) of $A$ and $B$.
(b) $\frac{8^{2} \times 8^{3}}{8^{4}}=2^{n}$

Find the value of $n$.

11 The diagram shows a right-angled triangle and a rectangle.


The area of the triangle is twice the area of the rectangle.
(i) Write down an equation for $x$.
(ii) Find the area of the rectangle.

Show clear algebraic working.

12 The grouped frequency table gives information about the times recorded for 20 runners in a 1500 metre race.

| Time $(\boldsymbol{t}$ seconds) | Frequency |
| :---: | :---: |
| $225<t \leqslant 230$ | 1 |
| $230<t \leqslant 235$ | 3 |
| $235<t \leqslant 240$ | 7 |
| $240<t \leqslant 245$ | 6 |
| $245<t \leqslant 250$ | 2 |
| $250<t \leqslant 255$ | 1 |

(a) Complete the cumulative frequency table.

| Time $(\boldsymbol{t}$ seconds) | Cumulative <br> frequency |
| :---: | :---: |
| $225<t \leqslant 230$ |  |
| $225<t \leqslant 235$ |  |
| $225<t \leqslant 240$ |  |
| $225<t \leqslant 245$ |  |
| $225<t \leqslant 250$ |  |
| $225<t \leqslant 255$ |  |

(b) On the grid, draw the cumulative frequency graph for your table.

(2)
(c) Use your graph to find an estimate for the median of the recorded times.
(2)
(Total for Question 12 is 5 marks)

13 The table shows information about the oil production, in barrels per day, of five countries during one year.

| Country | Oil production <br> (barrels per day) |
| :--- | :---: |
| India | $8.97 \times 10^{5}$ |
| Brazil | $2.63 \times 10^{6}$ |
| United States | $8.4 \times 10^{6}$ |
| Russia | $1.09 \times 10^{7}$ |
| Saudi Arabia | $9.9 \times 10^{6}$ |

(a) Which country had the highest oil production?
(b) Calculate the difference between the oil production of Brazil and the oil production of India. Give your answer in standard form.

During the same year, the oil production of California was $6.3 \times 10^{5}$ barrels per day.
(c) Work out the oil production of California as a proportion of the oil production of the United States.

14 Solve the simultaneous equations

$$
\begin{array}{r}
8 x-4 y=7 \\
12 x-8 y=6
\end{array}
$$

Show clear algebraic working.

$$
x=
$$

$$
y=
$$

## (Total for Question 14 is 3 marks)

15 Use algebra to show that the recurring decimal $0 . \dot{4} 1 \dot{7}=\frac{139}{333}$
$16 A B C D$ is a kite.


Diagram NOT accurately drawn
$A B=3 \mathrm{~cm}$
$B C=8 \mathrm{~cm}$
Angle $A B C=110^{\circ}$
Calculate the area of the kite $A B C D$.
Give your answer correct to 3 significant figures.

17 Two bags contain discs.
Bag A contains 12 discs.
5 of the discs are red, 6 are blue and 1 is white.
Bag B contains 25 discs.
$n$ of the discs are red and the rest are blue.
James takes at random a disc from Bag A.
Lucy takes at random a disc from Bag B.
Given that the probability that James and Lucy both take a red disc is $\frac{2}{15}$
(i) find the value of $n$, the number of red discs in Bag B.
$n=$
(ii) Hence calculate the probability that James and Lucy take discs of different colours.

18

$P Q R$ is a triangle.
The midpoint of $P Q$ is $W$.
$X$ is the point on $Q R$ such that $Q X: X R=2: 1$
$P R Y$ is a straight line.
$\overrightarrow{P W}=\mathbf{a} \overrightarrow{P R}=\mathbf{b}$
(a) Find, in terms of $\mathbf{a}$ and $\mathbf{b}$,
(i) $\overrightarrow{Q R}$
(ii) $\overrightarrow{Q X}$
(iii) $\overrightarrow{W X}$
$R$ is the midpoint of the straight line $P R Y$.
(b) Use a vector method to show that $W X Y$ is a straight line.

19 The diagram shows a circular pond, of radius $r$ metres, surrounded by a circular path.
The circular path has a constant width of 1.5 metres.


Diagram NOT
accurately drawn

The area of the path is $\frac{1}{10}$ the area of the pond.
(a) Show that $2 r^{2}-60 r-45=0$
(b) Calculate the area of the pond.

Show your working clearly.
Give your answer correct to 3 significant figures.

20 The diagram shows parts of the graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{g}(x)$.

(a) Find $g(0)$
(b) Find $\operatorname{gf}(-1)$
(2)
(c) Calculate an estimate for the gradient of the curve $y=\mathrm{f}(x)$ at the point on the curve where $x=3$

21 Correct to 2 significant figures, $a=58, b=28$ and $c=18$
Calculate the upper bound for the value of $\frac{a}{b-c}$
Show your working clearly.

22 Simplify fully $\frac{6 x^{2}+x-15}{12 x^{2}-27}$
Show clear algebraic working.

23

$A, B$ and $C$ are points on horizontal ground.
$B$ is due North of $A$ and $A B$ is 14 m .
$C$ is due East of $A$ and $A C$ is 25 m .
A vertical flagpole, $T X$, has its base at the point $X$ on $B C$ such that the angle $A X C$ is a right angle.

The height of the flagpole, $T X$, is 10 m .
Calculate the size of the angle of elevation of $T$ from $A$.
Give your answer correct to 1 decimal place.

