## International GCSE MATHEMATICS

## FORMULAE SHEET - HIGHER TIER



Volume of cone $=\frac{1}{3} \pi r^{2} h$
Curved surface area of cone $=\pi r l$


Volume of sphere $=\frac{4}{3} \pi r^{3}$
Surface area of sphere $=4 \pi r^{2}$



$$
\begin{aligned}
& \text { adj }=\text { hyp } \times \cos \theta \\
& \text { opp }=\text { hyp } \times \sin \theta \\
& \text { opp }=\operatorname{adj} \times \tan \theta
\end{aligned}
$$

or $\quad \sin \theta=\frac{\text { opp }}{\text { hyp }}$
$\cos \theta=\frac{\text { adj }}{\text { hyp }}$
$\tan \theta=\frac{\text { opp }}{\text { adj }}$
In any triangle $A B C$


Sine rule: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
Cosine rule: $a^{2}=b^{2}+c^{2}-2 b c \cos A$


Volume of prism $=$ area of cross section $\times$ length


Circumference of circle $=2 \pi r$
Area of circle $=\pi r^{2}$

## Area of a trapezium $=\frac{1}{2}(a+b) h$



The Quadratic Equation
The solutions of $a x^{2}+b x+c=0$, where $a \neq 0$, are given by
$x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

## Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.
You must write down all the stages in your working.
1 (a) Work out the value of $\frac{13.8 \times 6.5}{7+\sqrt{2}}$
Write down all the figures on your calculator display.
(b) Give your answer to part (a) correct to 3 significant figures.

2 Show that $\frac{4}{9} \div \frac{5}{6}=\frac{8}{15}$

3

(a) Describe fully the single transformation that maps shape $\mathbf{A}$ onto shape $\mathbf{B}$.
(b) On the grid, rotate shape $\mathbf{A} 90^{\circ}$ clockwise about the origin $O$. Label the new shape $\mathbf{C}$.

## Do NOT write in this space.

4 (a) Simplify $8 d \times 7 d$
(b) Expand 4(3e-5)
(c) Factorise $f^{2}-2 f$
(d) $H=g^{3}+6 g$

Work out the value of $H$ when $g=2$

5


Diagram NOT
accurately drawn

Calculate the length of $P Q$.
Give your answer correct to 3 significant figures.

6 The diagram shows an accurate scale drawing of part of the boundary of a field. The complete boundary of the field is in the shape of a quadrilateral $A B C D$.
$A B=300$ metres.
$B C=230$ metres.
Point $B$ is due north of point $C$.
The scale of the diagram is 1 cm to 50 metres.
The bearing of $D$ from $C$ is $260^{\circ}$
$A D=480$ metres.
Complete the scale drawing of the boundary of the field.
Mark the position of $D$.


230 m

C

7 (a) $A=\{\mathrm{p}, \mathrm{r}, \mathrm{a}, \mathrm{g}, \mathrm{u}, \mathrm{e}\}$
$B=\{\mathrm{p}, \mathrm{a}, \mathrm{r}, \mathrm{i}, \mathrm{s}\}$
$C=\{\mathrm{b}, \mathrm{u}, \mathrm{d}, \mathrm{a}, \mathrm{p}, \mathrm{e}, \mathrm{s}, \mathrm{t}\}$
List the members of the set
(i) $A \cap B$
(ii) $B \cup C$
(b) $D=\{\mathrm{r}, \mathrm{o}, \mathrm{m}, \mathrm{e}\}$
$E=\{1, \mathrm{i}, \mathrm{s}, \mathrm{b}, \mathrm{o}, \mathrm{n}\}$
$F=\{\mathrm{b}, \mathrm{e}, \mathrm{r}, \mathrm{l}, \mathrm{i}, \mathrm{n}\}$
Put one of the letters $D, E$ or $F$ in the box below to make the statement correct.

$$
A \cap \square=\varnothing
$$

Explain your answer.

## Do NOT write in this space.

8 (a) On the grid, draw the line with equation $x+2 y=8$ for values of $x$ from 0 to 9

(b) Show, by shading on the grid, the region defined by all three inequalities

$$
\begin{aligned}
& x+2 y \leqslant 8 \\
& x \geqslant 2 \\
& y \geqslant 1
\end{aligned}
$$

Label your region $\mathbf{R}$.


Diagram NOT
accurately drawn

The diagram shows a prism.
The cross-section of the prism is an isosceles triangle.
The lengths of the sides of the triangle are $13 \mathrm{~cm}, 13 \mathrm{~cm}$ and 10 cm .
The perpendicular height of the triangle is 12 cm .
The length of the prism is 8 cm .
Work out the total surface area of the prism.

10 Zara must take 5 tests.
Each test is out of 100
After 4 tests, her mean score is $64 \%$.
What score must Zara get in her 5th test to increase her mean score in all 5 tests to $70 \%$ ?

11 The table gives information about the speed, in $\mathrm{km} / \mathrm{h}$, of 180 vehicles passing a speed checkpoint.

| Speed <br> $(v \mathrm{~km} / \mathrm{h})$ | Frequency |
| :---: | :---: |
| $40<v \leqslant 50$ | 4 |
| $50<v \leqslant 60$ | 52 |
| $60<v \leqslant 70$ | 60 |
| $70<v \leqslant 80$ | 34 |
| $80<v \leqslant 90$ | 18 |
| $90<v \leqslant 100$ | 12 |

(a) Write down the modal class.
(b) Work out an estimate for the probability that the next vehicle passing the speed checkpoint will have a speed of $60 \mathrm{~km} / \mathrm{h}$ or less.
(c) Complete the cumulative frequency table.

| Speed <br> $(v \mathrm{~km} / \mathrm{h})$ | Cumulative <br> frequency |
| :---: | :---: |
| $40<v \leqslant 50$ |  |
| $40<v \leqslant 60$ |  |
| $40<v \leqslant 70$ |  |
| $40<v \leqslant 80$ |  |
| $40<v \leqslant 90$ |  |
| $40<v \leqslant 100$ |  |

(d) On the grid, draw a cumulative frequency graph for your table.

(e) The police decide to fine the driver of any vehicle passing the speed checkpoint at a speed of more than $84 \mathrm{~km} / \mathrm{h}$.
Use your graph to find an estimate for the number of drivers the police decide to fine. Show your method clearly.

12 (a) Helen's savings increased from $£ 155$ to $£ 167.40$
Work out the percentage increase in Helen's savings.
(b) Joe's savings increased by $4.5 \%$.

His savings are now $£ 125.40$
What were his savings before the increase?

## Do NOT write in this space.

13


Diagram NOT accurately drawn

The diagram shows a square $A B C D$ drawn inside a circle, centre $O$.
$A, B, C$ and $D$ are points on the circle.
The lengths of the sides of the square are 10 cm .
$A C$ is a diameter of the circle.
Calculate the circumference of the circle.
Give your answer correct to 3 significant figures.

14 A farmer has 180 metres of fencing.
With the 180 metres of fencing, he makes an enclosure divided into eight equal, rectangular pens.
The fencing is used for the perimeter of each pen.


The length of each pen is $x$ metres and the width of each pen is $y$ metres.
(a) (i) Show that $y=18-1.2 x$

The total area of the enclosure is $A \mathrm{~m}^{2}$.
(ii) Show that $A=144 x-9.6 x^{2}$
(b) Find $\frac{\mathrm{d} A}{\mathrm{~d} x}$
(c) Find the maximum value of $A$.

$$
A=
$$

15 The diagram shows two regular hexagons, $O A B C D E$ and $O F G H I J$.


Diagram NOT
accurately drawn
$O A F$ and $O E J$ are straight lines.
$O F=3 O A$.
The area of $O A B C D E$ is $4 \mathrm{~cm}^{2}$.
Calculate the area of the shaded region.
$16 A E C$ and $D E B$ are chords of a circle.

$A E=4 \mathrm{~cm}$.
$C E=9 \mathrm{~cm}$.
$D E=B E=x \mathrm{~cm}$.
Calculate the value of $x$.

$$
x=
$$

17 Make $x$ the subject of $y=\sqrt{\frac{2 x+1}{x-1}}$

18 A trapezium $A B C D$ has an area of $5 \sqrt{6} \mathrm{~cm}^{2}$.


Diagram NOT
accurately drawn
$A B=4 \mathrm{~cm}$.
$B C=\sqrt{3} \mathrm{~cm}$.
$D C=k \mathrm{~cm}$.
Calculate the value of $k$, giving your answer in the form $a \sqrt{b}-c$ where $a, b$ and $c$ are positive integers.
Show each step in your working.

Do NOT write in this space.

19 Rachael walks to school.
The distance to school is 2.8 km , correct to the nearest 0.1 km .
She walks at a speed of $5 \mathrm{~km} / \mathrm{h}$, correct to the nearest $\mathrm{km} / \mathrm{h}$.
Calculate the upper bound, in minutes, for the time Rachael takes to walk to school.
minutes

Do NOT write in this space.

20 Here are nine counters.
Each counter has a number on it.


The counters are turned over to hide their numbers and are then mixed up.
Susan takes at random a counter and turns it over to reveal its number.
She takes at random a second counter, from the remaining eight counters, and turns it over to reveal its number.
(a) Calculate the probability that the number 5 is on both of the two counters Susan takes.
(b) Calculate the probability that the sum of the numbers on the two counters Susan takes is divisible by 3

21 The function f is defined as $\mathrm{f}(x)=\frac{3}{4+x}$
(a) Find the value of $f(1)$
(b) State which value of $x$ must be excluded from any domain of f .

The function g is defined as $\mathrm{g}(x)=5+x$
(c) Given that $\mathrm{g}(a)=7$, find the value of $a$.

$$
a=
$$

(d) Calculate $\mathrm{fg}(1)$
(e) Find $\operatorname{fg}(x)$

Simplify your answer.

$$
\begin{equation*}
\operatorname{fg}(x)= \tag{2}
\end{equation*}
$$

22 The diagram shows a metal plate.


Diagram NOT
accurately drawn

The metal plate is made from a sector $O A B$ of a circle, centre $O$, and a triangle $O C B$.
Angle $A O B=65^{\circ}$ Angle $O C B=35^{\circ}$
$O A=O B=8 \mathrm{~cm}$.
$A O C$ is a straight line.
(a) Calculate the length of $B C$.

Give your answer correct to 3 significant figures.
cm
(b) Calculate the total area of the metal plate.

Give your answer correct to 3 significant figures.

23 Solve the equation $\frac{3}{(x+2)}+\frac{4}{(x-3)}=2$
Show clear algebraic working.

